Terrain-Like and Non-Jumping Graphs*

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Abstract

Let $G = (V, E)$ be a graph with n vertices. A *labeling* of (the vertices of) G is an injective function $\pi : V \to [n]$. We say that π is a *terrain-like labeling* of G if for any four vertices a, b, c, d such that $\pi[a] < \pi[b] < \pi[c] < \pi[d]$, if both $\{a, c\}$ and $\{b, d\}$ are in E, then so is $\{a, d\}$. The graph G is *terrain-like* if it has a terrain-like labeling. Similarly, π is a *non-jumping labeling* of G (Ahmed et al., 2017) if for any four vertices a, b, c, d such that $\pi[a] < \pi[b] < \pi[c] < \pi[d]$, if both $\{a, c\}$ and $\{b, d\}$ are in E, then so is $\{b, c\}$. The graph G is *non-jumping* if it has a non-jumping labeling (see Figure 1). In this paper we compare terrain-like graphs and non-jumping graphs, answering on the way a question raised by Ahmed et al. concerning the latter family.

1 Introduction

The family of terrain-like graphs was introduced by Ashur et al. [2], extending a manuscript of Katz [5]. Ashur et al. adapt the PTAS of Gibson et al. [4] for vertex guarding the vertices of x-monotone terrains, to obtain a PTAS for *minimum dominating set* (MDS) in terrain-like graphs. Then, by showing that the *visibility graphs* of *weakly-visible* polygons and terrains are terrain-like, they immediately obtain similar PTASs for guarding such polygons and terrains.

Ahmed et al. [1] defined the family of non-jumping graphs and proved that it is equivalent to the family of *monotone L-graphs* and thus admits a PTAS for MDS [3]. They showed that several well-known graph families, such as outerplanar graphs, convex bipartite graphs, and complete graphs, are subfamilies of non-jumping graphs and are therefore also monotone L-graphs. They also gave an example of a (non-planar) graph which is jumping (i.e. not non-jumping), providing a long and involved proof for it, and raised the question whether all planar graphs are non-jumping (and thus can be realized as monotone L-graphs).

Denote by $\mathcal{F}_{NJ}$ and $\mathcal{F}_{TL}$ the families of non-jumping and terrain-like graphs, respectively. The resemblance between the definitions of $\mathcal{F}_{NJ}$ and $\mathcal{F}_{TL}$, together with the fact that many of the graph families that were found to be non-jumping in [1] (including those mentioned above) are also terrain-like, raises the question what is the connection between them?

In this paper, we investigate the relation between these two graph families. First, we present a natural infinite family of graphs that are in $\mathcal{F}_{TL}$ but not in $\mathcal{F}_{NJ}$, and give a short and simple proof for it. Moreover, the smallest member of this family is a planar graph, implying that there exist planar graphs that cannot be realized as monotone L-graphs. Then, we present some basic properties of the terrain-like labeling function, and use them to prove that there exists an infinite family of graphs that are in $\mathcal{F}_{NJ}$ but not in $\mathcal{F}_{TL}$. Finally, we present a family of graphs which are not in $\mathcal{F}_{TL} \cup \mathcal{F}_{NJ}$.

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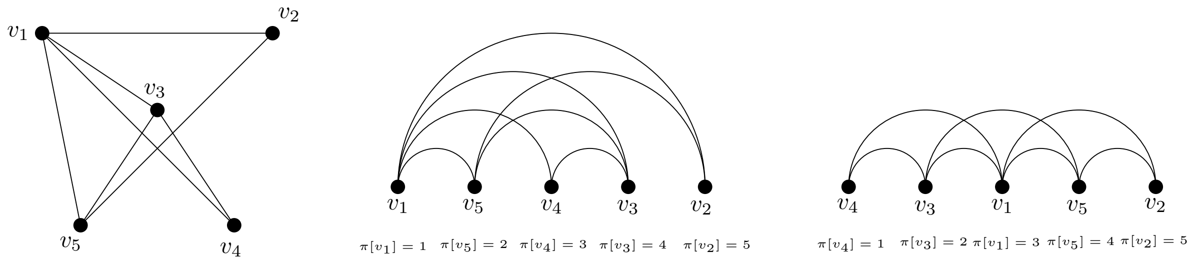

Figure 1 Left: The graph G. Center: A terrain-like (and jumping) labeling of G. Right: A non-jumping (and not terrain-like) labeling of G.

2 $\mathcal{F}_{TL}$ vs. $\mathcal{F}_{NJ}$

▶ **Theorem 2.1.** $\mathcal{F}_{TL} \not\subseteq \mathcal{F}_{NJ}$

Proof. Let $K_n = (V = \{v_1, v_2, \ldots, v_n\}, E)$ be the complete graph on n vertices. For $n \geq 6$, let $K_n^{-3} = (V, E \setminus \{e_1, e_2, e_3\})$, where e_1, e_2, e_3 are any three pairwise-disjoint edges in E. We show that for any $n \geq 6$, $K_n^{-3} \in \mathcal{F}_{TL} \setminus \mathcal{F}_{NJ}$. Assume w.l.o.g. that $e_1 = \{v_1, v_2\}$, $e_2 = \{v_3, v_4\}$, and $e_3 = \{v_5, v_6\}$.
$\underline{K_n^{-3} \in \mathcal{F}_{TL}}$: Consider the labeling $\pi[v_i] = i$. For any 4 vertices $v_{i_1}, v_{i_2}, v_{i_3}, v_{i_4}$ such that $i_1 < i_2 < i_3 < i_4$, we have $\{v_{i_1}, v_{i_4}\} \in E$ since $i_4 - i_1 \geq 3$; thus π is a terrain-like labeling.
$\underline{K_n^{-3} \notin \mathcal{F}_{NJ}}$: Assume by contradiction that $K_n^{-3} \in \mathcal{F}_{NJ}$, then there exists a non-jumping labeling π of K_n^{-3}. Assume w.l.o.g. that $\pi[v_1] < \pi[v_2]$. We claim that either $\pi[v_1] = 1$ or $\pi[v_2] = n$. Indeed, assume that $\pi[v_i] = 1$ for some $i \neq 1$ and $\pi[v_j] = n$ for some $j \neq 2$. Notice that $\{v_i, v_2\}$ and $\{v_1, v_j\}$ are edges of the graph, but $\{v_1, v_2\}$ is not an edge of the graph, so π is not a non-jumping labeling w.r.t. v_i, v_1, v_2, v_j — a contradiction. By symmetry, the above claim holds also for v_3, v_4 and for v_5, v_6, but then π is not an injective function. ◀

As a corollary, we get that not all planar graphs are non-jumping, thus answering the question raised by Ahmed et al. [1]. Indeed, it is easy to verify that K_6^{-3} is planar (see Figure 2).

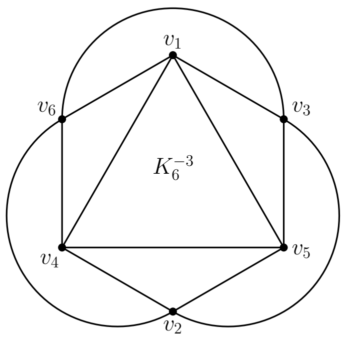

Figure 2 A planar embedding of K_6^{-3}.

2.1 Some properties of labeling functions

▶ **Observation 2.2.** *Let $G = (V, E)$ be a graph and let $H = (V', E')$ be an induced graph of G (i.e., $V' \subseteq V$ and $E' = \{\{u, v\} \mid u, v \in V', \{u, v\} \in E\}$). Let $\pi : V \to [|V|]$ be a terrain-like* *(resp., a non-jumping) labeling of G, and let $\pi' : V' \to [|V'|]$ be a labeling of H such that $\pi'[v_i] < \pi'[v_j]$ if and only if $\pi[v_i] < \pi[v_j]$. Then π' is a terrain-like (resp., a non-jumping) labeling of H.*

Denote by $P_n = (V, E)$ the path graph with n vertices, such that $V = \{v_1, \dots, v_n\}$ and $E = \{\{v_i, v_{i+1}\} \mid 1 \le i \le n-1\}$.

▶ **Lemma 2.3.** *Let π be a terrain-like labeling of P_n such that $\pi[v_1] = 1$ and $\pi[v_n] = n$, then $\pi[v_i] = i$ for $i = 1, \dots, n$.*

Proof. Let j be the largest index such that $\pi[v_i] = i$ for $i = 1, \dots, j$. If $j = n$ then we are done. Otherwise, $j \le n-3$ and $\pi[v_{j+1}] = k$, for some $j+1 < k < n$. Let l be the largest index such that $\pi[v_j] < \pi[v_l] < \pi[v_{j+1}]$, then $\pi[v_{j+1}] < \pi[v_{l+1}]$. But now π is not a terrain-like labeling w.r.t. $v_j, v_l, v_{j+1}, v_{l+1}$, since $\{v_j, v_{l+1}\} \notin E$, so j must be n. ◀

Denote by $C_n = (V, E)$ the cycle graph with n vertices, such that $V = \{v_1, v_2, \dots, v_n\}$ and $E = \{\{v_i, v_{i+1}\} \mid 1 \le i \le n-1\} \cup \{\{v_1, v_n\}\}$.

▶ **Lemma 2.4.** *Let π be a terrain-like (alternatively, a non-jumping) labeling of C_n such that $\pi[v_1] = 1$, then either $\pi[v_n] = n$ or $\pi[v_2] = n$.*

Proof. Assume that $\pi[v_2] < \pi[v_n]$. If $\pi[v_n] = n$ then we are done. Otherwise, let j be the smallest index such that $\pi[v_n] < \pi[v_j]$, and notice that $j \ge 3$. But now π is neither a terrain-like nor a non-jumping labeling w.r.t. v_1, v_{j-1}, v_n, v_j, since both $\{v_1, v_j\}$ and $\{v_{j-1}, v_n\}$ are not in E. The case $\pi[v_n] < \pi[v_2]$ is symmetric. ◀

▶ **Lemma 2.5.** *Let π be a terrain-like labeling of C_n. Assume w.l.o.g. that $\pi[v_1] = 1$ and $\pi[v_2] < \pi[v_n]$, then either:*

1. $\pi[v_1] < \pi[v_2] < \pi[v_3] < \cdots < \pi[v_{n-1}] < \pi[v_n]$, *or*
2. $\pi[v_1] < \pi[v_{n-1}] < \pi[v_{n-2}] < \cdots < \pi[v_2] < \pi[v_n]$.

Proof. By Lemma 2.4, $\pi[v_n] = n$, and thus for any $1 < i < n$ we have $\pi[v_1] < \pi[v_i] < \pi[v_n]$. First, we claim that if $\pi[v_2] < \pi[v_i]$ for some $3 \le i \le n-2$, then $\pi[v_2] < \pi[v_{i+1}]$. Indeed, if $\pi[v_1] < \pi[v_{i+1}] < \pi[v_2] < \pi[v_i]$ then we have $\{v_1, v_2\}, \{v_i, v_{i+1}\} \in E$ but $\{v_1, v_i\} \notin E$. Symmetrically, if $\pi[v_{n-1}] < \pi[v_i]$ for some $2 \le i \le n-3$, then $\pi[v_{n-1}] < \pi[v_{i+1}]$.

Secondly, we claim that if $\pi[v_i] < \pi[v_2]$ for some $3 \le i \le n-2$, then $\pi[v_{i+1}] < \pi[v_2]$. Indeed, if $\pi[v_1] < \pi[v_i] < \pi[v_2] < \pi[v_{i+1}]$ then we have $\{v_1, v_2\}, \{v_i, v_{i+1}\} \in E$ but $\{v_1, v_{i+1}\} \notin E$.

Therefore we can only have the following two cases:

1. If $\pi[v_2] < \pi[v_3] < \pi[v_n]$, then by the first claim we have $\pi[v_2] < \pi[v_j] < \pi[v_n]$ for $j = 3, \dots, n-1$. By Lemma 2.3 on the induced path $v_2, v_3, \dots, v_n$ we get that $\pi[v_1] < \pi[v_2] < \pi[v_3] < \cdots < \pi[v_{n-1}] < \pi[v_n]$.
2. If $\pi[v_1] < \pi[v_3] < \pi[v_2]$, then by the second claim we have $\pi[v_1] < \pi[v_j] < \pi[v_2]$ for $j = 3, \dots, n-1$, and, since $\pi[v_{n-1}] < \pi[2]$, by the first claim we have $\pi[v_{n-1}] < \pi[v_j]$ for $j = 2, \dots, n-2$. Again by Lemma 2.3 on the induced path $v_{n-1}, \dots, v_3, v_2$ we get that $\pi[v_1] < \pi[v_{n-1}] < \pi[v_{n-2}] < \cdots < \pi[v_2] < \pi[v_n]$.

◀

▶ **Theorem 2.6.** $\mathcal{F}_{NJ} \not\subseteq \mathcal{F}_{TL}$

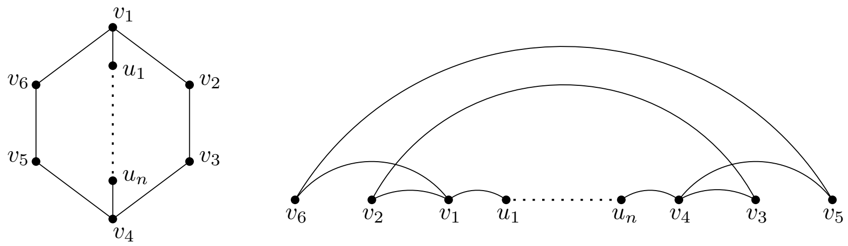

Figure 3 Left: The graph G. Right: A non-jumping labeling of G, i.e. $\pi[v_6] = 1, \pi[v_2] = 2, \pi[v_1] = 3, \pi[u_1] = 4, \ldots, \pi[u_n] = n+3, \pi[v_4] = n+4, \pi[v_3] = n+5, \pi[v_5] = n+6$.

Proof. Let C_6 be the cycle graph with vertex set $V = \{v_1, v_2, \ldots, v_6\}$, and P_n the path graph with vertex set $U = \{u_1, u_2, \ldots, u_n\}$, $n \geq 2$. Consider the graph $G = (V \cup U, E)$, where $E = E(C_6) \cup E(P_n) \cup \{\{v_1, u_1\}, \{v_4, u_n\}\}$. In other words, G contains an induced cycle on 6 vertices $v_1, v_2, \ldots, v_6$, and an induced path on $n+2$ vertices $v_1, u_1, u_2, \ldots, u_n, v_4$; see Figure 3 (left).

$G \in \mathcal{F}_{NJ}$

Figure 3 (right) shows a non-jumping labeling of G, so G is in $\mathcal{F}_{NJ}$.

$G \notin \mathcal{F}_{TL}$

Assume by contradiction that G is in $\mathcal{F}_{TL}$, then there exists a terrain-like labeling $\pi : V \cup U \to [n+6]$. Let $\pi_V : V \to [6]$ be a labeling such that $\pi_V[v_i] < \pi_V[v_j]$ if and only if $\pi[v_i] < \pi[v_j]$. Since C_6 is an induced cycle, Lemmas 2.4 and 2.5 can be applied to π_V. By Lemma 2.4, there must be an edge between the first and last vertex in the labeling π_V. Formally, if $\pi_V[v_i] = 1$ and $\pi_V[v_j] = 6$, then $\{v_i, v_j\} \in E$. There are 6 edges in C_6, so there are 6 possible choices of $e = \{v_i, v_j\}$, but we observe that the graph is symmetric for all the edges in $\{\{v_1, v_6\}, \{v_1, v_2\}, \{v_4, v_5\}, \{v_4, v_3\}\}$, and for all the edges in $\{\{v_5, v_6\}, \{v_2, v_3\}\}$. Thus, w.l.o.g. we only consider the following two cases: either $e = \{v_1, v_6\}$ or $e = \{v_5, v_6\}$. By Lemma 2.5 we have four cases for the labeling of V:

1. $\pi[v_1] < \pi[v_2] < \pi[v_3] < \pi[v_4] < \pi[v_5] < \pi[v_6]$
2. $\pi[v_1] < \pi[v_5] < \pi[v_4] < \pi[v_3] < \pi[v_2] < \pi[v_6]$
3. $\pi[v_6] < \pi[v_1] < \pi[v_2] < \pi[v_3] < \pi[v_4] < \pi[v_5]$
4. $\pi[v_6] < \pi[v_4] < \pi[v_3] < \pi[v_2] < \pi[v_1] < \pi[v_5]$

Cases 1 and 3: It is not hard to verify that either $\pi[v_3] < \pi[u_n] < \pi[v_4]$, or $\pi[v_4] < \pi[u_n] < \pi[v_5]$. Thus either $\pi[v_3] < \pi[u_i] < \pi[v_4]$ for all $1 \leq i \leq n$, or $\pi[v_4] < \pi[u_i] < \pi[v_5]$ for all $1 \leq i \leq n$. If $\pi[v_3] < \pi[u_1] < \pi[v_4]$, then the labeling $\pi[v_1] < \pi[v_3] < \pi[u_1] < \pi[v_4]$ contradicts the terrain-like property, and if $\pi[v_4] < \pi[u_1] < \pi[v_5]$, then the labeling $\pi[v_1] < \pi[v_4] < \pi[u_1] < \pi[v_5]$ is a contradiction.

Case 2: Again, we have either $\pi[v_4] < \pi[u_i] < \pi[v_3]$ for all $1 \leq i \leq n$, or $\pi[v_5] < \pi[u_i] < \pi[v_4]$ for all $1 \leq i \leq n$. If $\pi[v_4] < \pi[u_1] < \pi[v_3]$, then the labeling $\pi[v_1] < \pi[v_4] < \pi[u_1] < \pi[v_3]$ contradicts the terrain-like property, and if $\pi[v_5] < \pi[u_1] < \pi[v_4]$, then the labeling $\pi[v_1] < \pi[v_5] < \pi[u_1] < \pi[v_4]$ is a contradiction.

Case 4: Notice that either $\pi[v_6] < \pi[u_n] < \pi[v_4]$, or $\pi[v_4] < \pi[u_n] < \pi[v_3]$. Thus either $\pi[v_6] < \pi[u_i] < \pi[v_4]$ for all $1 \leq i \leq n$, or $\pi[v_4] < \pi[u_i] < \pi[v_3]$ for all $1 \leq i \leq n$. If

$\pi[v_6] < \pi[u_1] < \pi[v_4]$, then the labeling $\pi[u_1] < \pi[v_4] < \pi[v_1] < \pi[v_5]$ contradicts the terrain-like property, and if $\pi[v_4] < \pi[u_1] < \pi[v_3]$, then the labeling $\pi[v_4] < \pi[u_1] < \pi[v_3] < \pi[v_1]$ is a contradiction. ◀

Finally, does every graph belong either to $\mathcal{F}_{TL}$ or to $\mathcal{F}_{NJ}$? The answer is clearly no, since, in general, minimum dominating set is NP-hard to approximate within a factor of $\Omega(\log n)$ [6]. Nevertheless, it would be nice to see a concrete and simple example. Below, we present an infinite family of graphs which are neither in $\mathcal{F}_{TL}$ nor in $\mathcal{F}_{NJ}$.

The Harary graphs $H_{n,k}$ are k-connected graphs on n vertices, having the smallest possible number of edges. When n is even and k is odd, $H_{n,k}$ is defined as follows: $H_{n,k} = (V = \{v_0, ..., v_{n-1}\}, E_1 \cup E_2)$, where $E_1 = \{\{v_i, v_{i+j}\} | 1 \le j \le \lfloor \frac{k}{2} \rfloor, 0 \le i \le n-1\}$ and $E_2 = \{\{v_i, v_{i+\frac{n}{2}}\} | 0 \le i \le \frac{n}{2} - 1\}$ (where the addition is modulo n), see Figure 4.

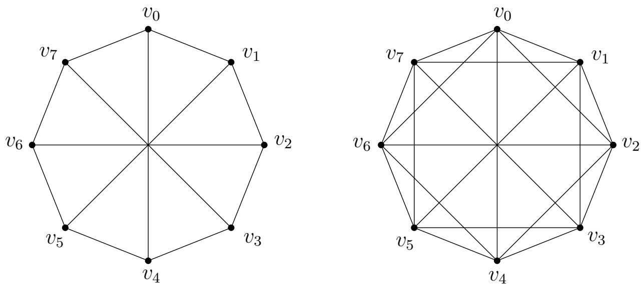

Figure 4 $H_{8,3}$ (left) and $H_{8,5}$ (right).

▶ **Theorem 2.7.** *For any $m \ge 4$, $H_{2m,3}$ is neither in $\mathcal{F}_{TL}$ nor in $\mathcal{F}_{NJ}$.*

Since we are interested in a simple example, we prove the theorem here only for $H_{8,3}$.

Proof. **(For $m = 4$)** Assume by contradiction that π is a non-jumping labeling of $H_{8,3}$, and assume w.l.o.g. that $\pi[v_0] = 1$. Since $C_1 = (v_0, v_1, v_2, v_3, v_4)$ and $C_2 = (v_0, v_4, v_5, v_6, v_7)$ are induced cycles, we can apply Lemma 2.4, and get 3 cases: (i) $\pi[v_4] = 8$, (ii) $\pi[v_1] = 8$, or (iii) $\pi[v_7] = 8$. Notice that (ii) and (iii) are symmetric cases, so we consider only cases (i) and (ii). We denote the labeling of C_1 by π_1 and the labeling of C_2 by π_2.

(i) Assume w.l.o.g. that $\pi[v_0] < \pi[v_1] < \pi[v_7] < \pi[v_4]$, then for any possible labeling of v_5 we get that π is not a non-jumping labeling: if $\pi[v_7] < \pi[v_5]$ then we have $\{v_0, v_7\}, \{v_1, v_5\} \in E$ but $\{v_1, v_7\} \notin E$, and if $\pi[v_5] < \pi[v_7]$ then we have $\{v_0, v_7\}, \{v_4, v_5\} \in E$ but $\{v_5, v_7\} \notin E$.

(ii) Notice that $\pi[v_0] < \pi[v_2] < \pi[v_4] < \pi[v_1]$ is not possible, so assume $\pi[v_0] < \pi[v_4] < \pi[v_2] < \pi[v_1]$. We notice that either $\pi_2[v_4] = 5$ or $\pi_2[v_7] = 5$. If $\pi_2[v_4] = 5$ then since $\pi_2[v_6] < 5$ we get that $\pi[v_6] < \pi[v_4]$, but then we have $\{v_0, v_4\}, \{v_6, v_2\} \in E$ but $\{v_4, v_6\} \notin E$. If $\pi_2[v_7] = 5$, then since $\pi_2[v_5] < 5$ we get that $\pi[v_5] < \pi[v_7]$, but then we have $\{v_0, v_7\}, \{v_5, v_1\} \in E$ but $\{v_5, v_7\} \notin E$.

Now assume by contradiction that π is a terrain-like labeling of $H_{8,3}$, and assume w.l.o.g. that $\pi[v_0] = 1$. Again by applying Lemma 2.4 we have four cases: (i) $1 = \pi[v_0] < \pi[v_1] < \pi[v_2] < \pi[v_3] < \pi[v_4] = 8$, (ii) $1 = \pi[v_0] < \pi[v_3] < \pi[v_2] < \pi[v_1] < \pi[v_4] = 8$, (iii) $1 = \pi[v_0] < \pi[v_4] < \pi[v_3] < \pi[v_2] < \pi[v_1] = 8$, (iv) $1 = \pi[v_0] < \pi[v_2] < \pi[v_3] < \pi[v_4] < \pi[v_1] = 8$.

(i) We first get that $\pi[v_0] < \pi[v_5] < \pi[v_1]$ since any other labeling results in a contradiction, and then any labeling of v_6 given $1 = \pi[v_0] < \pi[v_5] < \pi[v_1] < \pi[v_2] < \pi[v_3] < \pi[v_4] = 8$ is impossible.

(ii) There are 2 possibilities for labeling v_5: either $\pi[v_0] < \pi[v_5] < \pi[v_3]$ or $\pi[v_1] < \pi[v_5] < \pi[v_4]$. If $\pi[v_0] < \pi[v_5] < \pi[v_3]$ then we have $\pi[v_2] < \pi[v_6] < \pi[v_1]$ and no possible labeling for v_7. If $\pi[v_1] < \pi[v_5] < \pi[v_4]$ then there is no possible labeling for v_6.

(iii) We first get that $\pi[v_0] < \pi[v_5] < \pi[v_4]$ since any other labeling results in a contradiction, and then any labeling of v_6 given $1 = \pi[v_0] < \pi[v_5] < \pi[v_4] < \pi[v_3] < \pi[v_2] < \pi[v_1] = 8$ is impossible.

(iv) There are 2 possibilities for labeling v_5: either $\pi[v_0] < \pi[v_5] < \pi[v_2]$ or $\pi[v_4] < \pi[v_5] < \pi[v_1]$. If $\pi[v_0] < \pi[v_5] < \pi[v_2]$ then we have $\pi[v_5] < \pi[v_6] < \pi[v_2]$ and no possible labeling for v_7. If $\pi[v_4] < \pi[v_5] < \pi[v_1]$ then there is no possible labeling for v_6.

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